Thesis abstract

Topics in the theory of enriched accessible categories

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The aim of this thesis is to further develop the theory of accessible categories in the enriched context. We study and compare the two notions of *accessible* and *conically accessible* V-categories, both arising as free cocompletions of small V-categories: the former under flat-weighted colimits and the latter under filtered colimits. These two notions are not the same in general, however we show that they coincide for many significant bases of enrichment such as **Cat** and **SSet**, and differ just by Cauchy completeness for many algebraic examples including **Ab**, R-**Mod** and **GAb**. We then provide new characterization theorems for these by considering some notions of *virtual orthogonality* and *virtual reflectivity* which generalize the usual reflectivity and orthogonality conditions for locally presentable categories. The word virtual refers to the fact that the reflectivity and orthogonality conditions are given in the free completion of the V-category involved under small limits, instead of the V-category itself. We then prove that the 2-category of accessible V-categories, accessible V-functors, and V-natural transformations has all flexible limits. In the final chapters we study, characterize, and provide duality theorems in the setting of accessible V-categories with limits of a specified class Ψ; in this context, instead of the free completion under small limits, we consider "free completions" under a specific type of colimits $\mathfrak{C}$ for which, in particular, $\mathfrak{C}$-colimits commute in V with Ψ-limits. This allows us to capture the theories of weakly locally presentable, locally multi-presentable, locally poly-presentable, and accessible categories as instances of the same general framework.

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