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A marriage made in heaven — mathematics and computers

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Abstract

Professor Ian Sloan AO PhD FAA FRSN is the immediate past President of the Royal Society of New South Wales. This is his presidential paper.

Introduction

In this non-technical article I celebrate the extraordinary contribution to the world made by mathematics in combination with modern computing over the past six decades or more. Because my working career as a research mathematician and physicist has spanned roughly the same period, I have experienced many of these extraordinary developments at first hand, and can share some of those experiences with you.

Of course, mathematics itself traces back millennia rather than decades, and indeed was used to create computational tools, even by the ancient Greeks.[1] But until the coming of electronic computers all calculations were extraordinarily arduous, time consuming, and prone to error.

As an undergraduate student at the University of Melbourne in the late 1950s, I recall a take-home assignment on "numerical mathematics," for which we were to do the calculations on a hand-operated calculating machine, the famous "Brunsviga." These machines were engineering masterpieces, with ten or so levers on the front and a solid handle at the end for turning a cylinder. Let me explain how to multiply two decimal numbers, say 1.2468 by 5.234: one would enter the first number by setting five levers to the appropriate level 1, 2, 4, 6, 8 respectively (that's the easy part), and then rotate the handle 5 times for the leading digit of the multiplier, then shift something and rotate the handle 2 times for the next digit, then shift and rotate 3 times for the next digit, then finally shift and rotate 4 times for the last digit. (You can forget about the decimal point: you can see easily where it should go in the product.) If the number of turns was to be more than five, say 7, to save time one would rotate backwards 3 turns. All of this you could learn to do in maybe 10 seconds, but you certainly had to concentrate.

A more substantial criticism was that the methods used, and the problems tackled, were inevitably extremely limited, and restricted to simple and frankly boring problems. The typical problem was to compute the value of a certain function for some given input number (to be definite, let's say the logarithm to the base 10 of the given input number; but if you don't remember what a logarithm is, it doesn't matter in the slightest). In those days every school and university had many tables of logarithms of numbers, given at equally spaced inter-

1 https://en.wikipedia.org/wiki/Antikythera_mechanism [Ed.]

vals. But what about if you wanted the logarithm of a number (such as 1.2468) that was in between two entries in the table? In that case one would need to "interpolate" to obtain the number. That would require (depending on the accuracy you want, and the spacing used in the table) some number of additions and multiplications on your Brunsviga calculator.

Thank goodness, all that pain has gone. Every scientific calculator these days has logarithms (and many other functions) built in — all that underlying mathematical work is now built into the software. Now we can focus on more interesting problems.

In this article I concentrate on problems in just five areas of application; but, believe me, there are a multitude of others.

Weather forecasting

Have you noticed that the quality of the weather forecasts we look at each day has improved greatly over recent years? In part this is because of better observational data over land and over oceans. But it is also because of better mathematical models, better mathematical techniques, and of course better computers.

The scientific approach to weather forecasting is often attributed to Lewis Fry Richardson, a British applied mathematician working in the 1920s.[2] Our weather is determined by the physics and chemistry of what happens in the atmosphere, which is the thin film of oxygen, nitrogen, water vapour, carbon dioxide and other gases that covers the first few tens of kilometres above us. The temperature, wind, humidity and other quantities within this zone are governed by mathematical equations. (I hope that this is known to all, but I suspect not.) It is the working out of the physical systems described by these equations that determines the weather. What makes it all so difficult is that what happens at one point (say a pressure or temperature change) affects what happens at nearby places, and at adjacent times. And the pressure and temperature vary not only at places on the ground, but also at different heights above the ground. Everything is connected. To work out the consequences of those equations, and all that connectivity, supercomputers are needed.

I was struck by this sentence from the NASA website mentioned above: "Despite the advances made by Richardson, it took him, working alone, several months to produce a wildly inaccurate six-hour forecast for an area near Munich, Germany." Several months of computation to produce an unusable 6-hour forecast. That's exactly the point: that with the equipment he had available at the time (perhaps a Brunsviga calculator) there was absolutely no way of obtaining a useful forecast in real time. That's what has changed: not only has forecasting improved enormously, but also a forecast can be provided speedily enough to be useful.

In his book *Weather Prediction by Numerical Process*, published in 1922, Richardson evidently realised that real-time forecasting would require more resources: he estimated that the job could be done if he could have 64,000 assistants all in the one room.

Nowadays the Australian Bureau of Meteorology owns immensely powerful supercomputers, used every day for its forecasts. It also employs many scientists, who work continually to improve the models and the

2 See https://earthobservatory.nasa.gov/features/WxForecasting/wx3.php

science incorporated in those models. Some also work to improve the approximation schemes needed to restrict the mathematical equations (which are about continuous temperature or wind fields) to the discrete space and time grids used on the computer.

Statistics (another branch of the mathematical sciences) also plays a big role at the Bureau. One aspect of forecasting with strong statistical implications is what is known as "data assimilation." This describes the process of revising a forecast to take account of new information: perhaps rain was predicted at noon at a certain spot, but in fact it has already rained at 8 AM; how should the forecast be revised? And what is the uncertainty in the revised forecast? (Have you noticed that BoM forecasts now come with probabilities, for example the probability of rain over successive three-hour periods?) And of course all must be done quickly, and automatically, or else the forecast (like Richardson's) will be unuseable.

Computing rocket and spacecraft trajectories

The story of the computation of trajectories of rockets, like weather forecasting, traces back well before the age of electronic computers. Indeed, the first "computers" were not electronic, but human. The early history of the Jet Propulsion Laboratory (which later morphed into NASA) is told in Lutz (2016).

From that source, "What's often not known is that all the early rocket experiments and later missions to the moon and beyond wouldn't have been possible without a team at JPL known as the human 'computers.' Most of these human computers were women, who either had degrees in mathematics or were simply very good at mathematics. Over the course of time, these women not only performed hundreds of thousands of mathematical calculations crucial to the U.S. space program, but also eventually became some of the first computer programmers at NASA." The recent movie "Hidden Figures" is a dramatic representation of the contribution of these early "computers."

It is said that the human computers worked with pencil and paper. I suspect they also had the benefit of Brunsviga calculators and early electro-mechanical computers. But however they did it, they did remarkably well with old technology, and old mathematical tools.

This is perhaps a good moment for me to mention the importance in the old days of error control. When every calculation (and every single addition, multiplication or division) was done by the fallible hand of a single human, the possibilities of error were immense. For that reason, much of the literature in those days was devoted to the detection and correction of errors. As a young researcher I used to look regularly at the journal *Mathematical Tables and Other Aids to Computation* because its main role was to report errors in published tables. That was important in case one had to rely on a table that might contain errors. That's another painful task that has disappeared. (Some of those "errors" in tables were said to be deliberate, designed to flush out plagiarists.)

But I want to emphasise that error control in the broader sense is still very important. In my early days in research, working in physics, I came to the sad conclusion that around half the published papers contained significant errors, either mathematical or

computational (the latter meaning that the published numbers were not correct, the former that even the formulas were not correct). If this is less true now that I work mainly in mathematics, it is possibly because publication in mathematics is so slow that there is more time to correct errors.

Climate change

The Royal Society of London, the world's oldest scientific body, recently issued a call[3] for the creation of a multinational supercomputer centre, to provide climate modelling facilities beyond the capacity of any one nation to sustain, which will develop models on an unprecedentedly fine scale. The call notes that to double the precision of present modelling, a tenfold increase in computing power is required. This makes sense if you consider that to improve the resolution from 20 km to 10 km on the earth's surface requires four times (or 2 squared) as much data (because the surface is 2-dimensional); and to improve the resolution also in the vertical direction by a factor of two requires a further factor or two; and to halve the time step needs yet another factor of two. And that does not consider the extra processing power required for that much connectivity and so much more data.

Interestingly, the Royal Society does not say much about mathematics in its document. Why not? Because the writers of the document along with the earth scientists know full well that every aspect of the underlying models is expressed in terms of mathematics. Here is one quote: "These laws, represented by mathematical equations, have to be solved using sophisticated numerical techniques." Yes indeed.

Extreme bushfires

In a recent lecture at the Royal Society of NSW I learnt about "Extreme Bushfires and the Age of Violent Pyroconvection."[4] In brief, extreme bushfires (such as those we saw in 2019–20 in Eastern Australia) are bushfires that are violent enough to create their own weather. The reason we are coming into the age of extreme bushfires and "violent pyroconvection" is of course climate change.

I learnt that most bushfires are not in this sense extreme, and for those that become extreme the damage often happens during very short but violent episodes. Extreme bushfires are hard to predict, and even harder to manage. For an insight into how such events can be modelled and understood, see the excellent lecture by Jason Sharples FRSN in the YouTube video recording.[5]

My interest here is in an aspect little mentioned in the lecture, the hidden cooperating giant fields of mathematics and computing. As before, mathematics is everywhere when the physics and chemistry of the atmosphere and the environment are involved, and under extreme conditions they make a highly challenging and volatile cocktail. What about computers? The website of National Computational Infrastructure (which is Australia's national supercomputing centre) describes a ten-year partnership on Extreme Bushfires between Sharples'

3 https://policycommons.net/artifacts/1724705/next-generation-climate-models/2456354/

4 A paper based on this talk is in preparation for publishing here. [Ed.]

5 https://www.youtube.com/watch?v=cTRXkM_z-S8

group and NCI, reported in "Protecting lives and property from extreme bushfire."[6] This website describes supercomputer modelling that has the ultimate aim of being able to predict extreme bushfire events.

But prediction is not the end of the game, not if (like Lewis Richardson's weather forecasting efforts a century ago) the result arrives after the crisis has passed. The key to achieving the necessary "faster-than-real-time modelling" has many facets. As explained to me by Professor Sharples: the need is for

1. Understanding the fundamental processes driving extreme bushfire development
2. Computational models ... [that] inform ... the development of simplified "proxy" mathematical models
3. "Simplified" mathematical models sometimes require specific computational methods to deal with sources of numerical instability
4. Drawing upon fundamental mathematical theorems
5. The potential for Artificial Intelligence to support prediction of extreme bushfires.

These headings are perhaps enough to hint at the many challenges involved in the future design of an "app" that will give an authentic prediction in real time of an extreme bushfire event. Mathematics and computers need to work together!

Quantum physics

Never fear: I am not about to teach you about quantum physics. But I do want to say that the computations that are trying to explore the fundamental nature of matter (for example under such headings as string theory and lattice gauge theory) are some of the most challenging computations undertaken anywhere. It is common worldwide that non-military supercomputers spend much of their time doing lightning-fast calculations of enormous complexity on problems coming from quantum physics. Often the underlying approximation scheme (necessary to convert the mathematical equations to computer code) uses the so-called Monte Carlo method,[7] which as the name suggests relies on choosing numbers randomly. With colleagues in Germany and Australia I have had some recent involvement in this quantum physics activity, the key being a large interest in my group at UNSW in methods similar to Monte Carlo (so-called Quasi Monte Carlo methods), which however aim to be "better than random." This is one way of trying to achieve ever faster solutions of ever larger problems.

At the beginning of my research career, as a PhD student at the University of London, my project involved working out new ideas for calculating how a beam of electrons would be scattered by a collection of hydrogen atoms — an important question for astrophysics, and one not accessible to earthbound experiments (because on earth hydrogen atoms are invariably joined to other atoms to form molecules).

If I had arrived in London much earlier, then I would have spent perhaps six months sitting with a Brunsviga calculator (I don't remember seeing any electric calculators), doing unbelievably repetitive and tedious calculations every day. By the greatest good

6 https://nci.org.au/research/research-highlights/protecting-lives-and-property-extreme-bushfire

7 See Marks R.E. (2014) "Monte Carlo," in *The Palgrave Encyclopædia of Strategic Management*, edited by David Teece and Mie Augier, London: Palgrave. [Ed.]

fortune, I happened to arrive when the University of London was soon to have access to an advanced electronic computer. This was the "Atlas" computer built by the Ferranti company and Manchester University, said to be the fastest computer in the world when it came online in 1962.[8] I suppose that it had roughly the capacity and computational power of a modern smart phone, but at time the advance was tremendous. The Atlas wasn't so easy to use: the Autocode program had to be typed on easily torn paper tape, so one became an expert at splicing paper tape. But at least I was saved (by not being in Manchester) from the fate of many beginning PhD students in the early days of electronic computers, of having to nurse the computer overnight, in case of a "bug" in the program or the machinery.

I count it as the greatest of good fortune that I came online at more or less at the same time as modern computers.

Conclusion

In the modern world of science and technology, mathematics in a marriage with computers is so often providing the answer. The role of mathematics is very often unmentioned: mathematics is often the hidden secret agent. Do please remember that "computer model" almost always means "mathematical model captured in computer code."

8 See https://en.wikipedia.org/wiki/Atlas_(computer)

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