Journal & Proceedings of the Royal Society of New South Wales, vol. 150, part 2, 2017, pp. 188–194. ISSN 0035-9173/17/020188-07

P. A. M. Dirac and the Maverick Mathematician

Ann Moyal

Emeritus Fellow, ANU, Canberra, Australia

Email: moyal.ann@gmail.com

Abstract

Historian of science Ann Moyal recounts the story of a singular correspondence between the great British physicist, P. A. M. Dirac, at Cambridge, and J. E. Moyal, then a scientist from outside academia working at the de Havilland Aircraft Company in Britain (later an academic in Australia), on the question of a statistical basis for quantum mechanics. A David and Goliath saga, it marks a paradigmatic study in the history of quantum physics.

P. A. M. Dirac (1902–1984) is a preeminent name in scientific history. In 1962 it was my privilege to acquire a set of the letters he exchanged with the then young mathematician, José Enrique Moyal (1910–1998), for the Basser Library of the Australian Academy of Science, inaugurated as a centre for the archives of the history of Australian science. This is the only manuscript correspondence of Dirac (known to colleagues as a very reluctant correspondent) held in Australia. The twenty-four letters exchanged between the two men in 1944–46 at the height of war offer an important and little known case study — a paradigmatic study — in the history of physics.

The two correspondents, despite a common engineering component, came from very different backgrounds, José Enrique Moyal (who would fetch up as Reader in Statistics at the Australian National University in 1958–1965) was born in 1910 in Jerusalem to a Jewish lawyer father and a French mother and was educated at the local Herzliya Hebrew Gymnasium in Tel Aviv. From there, unmentored, he made his way in 1927 to Magdalene College, Cambridge, to study mathematics, but facing unsupported university costs, he moved to the school of engineering at the Institut d'Electrotechnique in Grenoble, enrolling subsequently at the Ecole Supérieure d'Electricité in Paris. Trained as a civil engineer, Moyal worked for a period in Tel Aviv but returned to Paris in 1937, where his exposure to such foundation works as Georges Darmois's *Statistique Mathématique* and A. N. Kolmogorov's *Foundations of the Theory of Probability* introduced him to a knowledge of pioneering European studies of stochastic processes. In 1939 he added theoretical physics focused on the foundations of quantum theory at the Institut Henri Poincaré at the University of Paris. After secret wartime work in Paris with the French Ministry of Air, and the German invasion of France, Moyal escaped to Britain where he was posted by C. P. Snow, then in charge of Scientific Manpower, for wartime research at the de Havilland Aircraft Company at Hatfield.

Paul Adrien Maurice Dirac, by contrast, was born in Bristol in 1902, took a degree at Bristol University in engineering and graduated in 1923 with first-class honours in mathematics. With initial published research in statistical mechanics, he gained his Ph.D. in quantum mechanics in 1926, from which his classic work, *The Principles of Quantum Mechanics* (Dirac, 1930–58), led him to election to the Royal Society of London in 1930, appointment as Lucasian Professor at Cambridge in 1932 (a post he held for 47 years), and a Nobel Prize, which he shared with Erwin Schrödinger in 1933. Widely regarded as the high priest of British physics, Dirac had established the standard theory of quantum physics, his book became the bible in the field, and he was assured that his methodology was correct and his theory complete.

It was to this pre-eminent but reclusive scholar, an academic known for ignoring the work of upcoming younger men (Farmelo, 2009), that J. E. Moyal, a British citizen of the Mandated Territory of Palestine, a researcher from outside academia, applied by letter on 18 February 1944 with his controversial idea of the possibilities of a statistical basis for quantum mechanics and a method to make the connection of classical mechanics to quantum mechanics in phase space. He found the professor in receptive mood. "I should be glad to meet you any weekend," Dirac replied on 22 February, "so choose any weekend you like." Their meeting on 11 March 1944 at Dirac's house in Cavendish Avenue, Cambridge, however, apparently brought the professor little joy. As his biographer, Helge Kragh, writes, Dirac "did not consider the probabilistic interpretation as something inherent in the quantum mechanical formulations" (Kragh, 1999).

But, undeterred, on 26 June 1944 Moyal returned to the task.

> On thinking over the objection you raised when I last saw you to my statistical treatment of quantum mechanics, [he wrote] it has occurred to me that the difficulties are chiefly a question of interpretation… As I explained in my paper, I consider the form I obtained for the phase-space distribution $F(p,q)$ as in a way of extension, or rather an exact form of Heisenberg's principle of uncertainty.

Sketching out his mathematical formulation carefully, he posited that, in fact, the dynamical problems the two had discussed offered "one case where the methods I have outlined may have advantage over the usual method." The theory led, in Moyal's view, to the distribution of phase space and also to correlations at two instants of time where, he suggested, "there is a possibility that it may lead to experimental verification in the field of electron and molecular beams." "Another field where I think the theory may be of some value," he added, "is in the study of statistical assemblies, since it leads to phase-space distributions for p and q, for Fermi-Dirac and Bose-Einstein assemblies."

Dirac, however, remained silent. When he wrote again some nine months later, on 19 March 1945, he had slightly ameliorated his negative view and indicated that work he had been doing had caused him to think there might be "a limited region of validity for the use of a joint probability distribution" in Moyal's work. Seeking a copy of the paper, he conceded, "I may get a more favourable opinion of it this time." Responding on 22 March 1945, Moyal referred Dirac to Maurice Bartlett[1] who had "worked out a new and improved method of obtaining a joint distribution" and noted that, in collaboration with Bartlett, he himself had also "carried further the treatment of the harmonic oscillator in phase space" (Bartlett and Moyal, 1949). "I also [he added] have been considering applications to statistical

[1] Maurice Bartlett had studied statistical mechanics with R. H. Fowler and physics with Dirac at Cambridge. He became a leading figure in Britain in statistical mechanics and probability theory.

mechanics which, since they require distribution in phase space, would seem to offer an obvious field for the theory."

Dirac's reply a month later, on 20 April 1945 (for both men were engaged in demanding wartime work, Moyal researching the mathematical character of complex systems, electronic instrumentation and the theory and practice of space vibrations and waves needed in the wartime aircraft industry, and Dirac importantly for the government on uranium separation relating to the construction of atomic bombs), was less than encouraging. He did not believe that the phase-space approach offered anything significant outside his own established formulation. Clearly satisfied with this dismissal and committed to his own interpretation of non-commuting observables in the paper he was preparing for *Reviews of Modern Physics*, he proposed to refer to Moyal's work in broad terms. "The possibility of setting up a probability for non-commuting observables in quantum mechanics to have specified values," he set down, "has been previously considered by J. E. Moyal… but its region of applicability is rather restricted and it does not seem to be connected with a general theory of functions like the present one."

Moyal's letter in reply on 29 April 1945 was robust. "I do not think that your reference to my work gives a correct description of it," he wrote (for Dirac, it appears, was confusing commuting p and q variables with non-commuting operators, P and Q). And unintimidated by his contester, he went on: "This would perhaps not matter a great deal if my work was already published, since readers could then refer to the original. I have not however been able so far to arrange for its publication, due largely as you will no doubt remember, to your veto which made the late Professor Fowler hesitate about presenting it to the Royal Society." As an outsider pinning his hopes of a research career on his research achievement, Moyal's frustration was real, since he believed that his reply to the dominant physicist had presented his reservations satisfactorily. "The papers you have seen," he now told Dirac, "represent my first real effort at research in pure mathematics and theoretical physics; I was hoping that their publication would eventually enable me to transfer my activities entirely from the field of engineering and applied physics to that of pure science. …Failure to obtain publication has forced me to adjourn such plans *sine die*."

The mathematician from outside academia had run up against an entrenched paradigm. As the most esteemed figure of quantum mechanics in Britain, Dirac's position within the discipline was set in stone. At Cambridge, he had always conducted his research on his own, in marked contrast to his eminent European colleagues who enjoyed the advantage of both formal and informal collaboration. Moreover, from his earliest endeavours, he was introspective and tenacious in his confidence in his own views. With sixty-four published papers behind him in 1945 and his foundation book, he appeared, as both Kragh and the distinguished Australian mathematician, Alan McIntosh, characterised him, as "intellectually incapable of, and unwilling to give ground" (McIntosh, 2003).

The David and Goliath struggle would continue across two more months. In his letters of 11 and 18 May 1945, Dirac again resisted Moyal's position and attempted to show that his argument was trivially wrong. But, stirred perhaps by Moyal's charge over publication, he went so far as to suggest: "I would be willing to help you publish if you would change it [the presentation] so that it does not contain any general statements which I think to be wrong."

Such surrender was not acceptable to the independent Moyal. His forthrightness owed something to his 'Israeli' background. He would not be cowed. Born in Palestine in the last decade of the Ottoman Empire, Moyal belonged to no particular nation but, as the region's history unfolded, he deemed himself an 'Israeli'. "Summarizing," he concluded in his letter to Dirac of 15 May 1945, "I think it would be fair to say that my paper gives a derivation of classical quantum mechanics on a purely statistical basis (plus Newtonian mechanics) which is equivalent to the standard matrix theory… and furthermore that it shows the consequences such a theory entails with regards to the problem of determinism, probability distributions, fluctuations, quantum statistics." He would affirm his position even more firmly in his subsequent letter of 26 May 1945:

> I don't think [it read] your remark on [my] getting the right answer 'by borrowing sufficient results from the ordinary quantum theory' quite fair. In so far as my theory is equivalent to the ordinary theory, it leads to the same eigenvalues for the mean of the energy, as I have shown in my paper. In order to prove an inherent inconsistency in my theory one would have to show that the method you use follows necessarily from my basic postulates, and this is not the case. My method on the other hand is based on a theory for statistical assemblies resulting from these postulates. As such it is quite consistent with the rest of the theory, and also appears to lead to correct results.

To little avail. Despite Moyal's objection to its contents, Dirac made no change in his original reference to his discussant's work in his *Reviews of Modern Physics* paper of 1945 although, in a rare reference to a contemporary researcher, he went so far as to allow: "This work is not yet published. I am indebted to J. E. Moyal for letting me see the manuscript." Dirac, however, had made one positive recommendation. If Moyal wrote the quantum theory part of his work in a separate paper, he himself could communicate it to a scientific journal. More controversial communication lay ahead, but, in July 1945, Moyal was notifying Dirac, that, as suggested, he was "rewriting the part of my paper on quantum mechanics as a separate paper."

Moyal's, 'Quantum Mechanics as a Statistical Theory' was at last submitted to the Cambridge Philosophical Society from his first academic post at the Department of Mathematical Physics, Queen's University, Belfast, in November 1947 and published in their *Proceedings* in 1949 (Moyal, 1949a). A second section of his manuscript was published as his paper, 'Stochastic Processes and Statistical Physics', in the *Journal of the Royal Statistical Society* (Moyal, 1949b) that same year. But as he affirmed privately in later years, "my first paper really contained all the essentials of the formalism, the version of quantum which is an equivalent of older mechanics" (Moyal A., 1979).

'Quantum Mechanics as a Statistical Theory' proved to be a research contribution far ahead of its time. Received initially by a small range of researchers in quantum fields, it made quiet headway while the fundamental formalism it presented in the 'Moyal bracket', the 'Moyal formula' and 'Moyal plane' (phrases that indicated non-commutativity) flowed into the lexicon of physics from the 1960s, gathering range and currency as the international research community grew. It was not, however, until the new century that the paper burst into high prominence and came to underlie an explosion of research in quantum physics and related fields that stretched from string theory, atomic and molecular systems and quantum chaos to optics, biology, mathematical theory, and an array of cascading computational and technological developments.

Moyal's own career as an academic had moved on from his appointment as a lecturer at Queen's University, Belfast, in 1946, to a rich period from 1950 in the Department of Mathematics at Manchester University, and brought him to the Australian National University in 1958. Headhunted in 1965 by America's leading national laboratory for the peaceful uses of atomic energy, Argonne National Laboratory, for his work on nuclear physics, probability and stochastic processes, he returned to Australia as a senior professor in the Department of Mathematics at Macquarie University in 1972. A researcher across three fields of mathematics, statistics, and quantum physics, his publications ran to thirty-six major papers but, in the award of a Doctor of Science *honoris causa* by the ANU in 1997, he was pronounced one of the most original thinkers in twentieth-century Australia. Although aware in his retirement of a significant flow-on of his work in quantum mechanics, he did not live to see the final effect of its range and power. Joe Moyal died in May 1998 at the age of 87. He knew, however, that he had fought a singular fight and preserved the correspondence for posterity.[2]

For my part, I had married Joe Moyal in 1963 and, as a historian of Australian science and technology, was alerted after his death to the accelerating impact of his 'Quantum Mechanics as a Statistical Theory' by his former younger colleagues at Macquarie University. I followed its growing presence in the scientific citations at the Web of Science. After five decades, in 2001 they sat at 69 hits (scientists are exultant if they reach the 100 mark), but in 2003 the citations soared to 980, to 1,220 in 2005 and, by 2006 had reached 1,245. In March 2017 citations of the paper at the Web of Science sat at 1,983, and at Google Scholar at 3,129. It was, as Peter Medawar once famously defined the core accomplishment of science, "a seminal theory that had come to stretch far beyond its own creative era and to emerge as a general statement of steadily increasing explanatory power and compass" (Medawar, 1969).

Steered by these colleagues, I would carry the story forward. Alan McIntosh, FAA, who had become head of the Centre for Mathematics and its Applications at the Australian National University, having read a copy of the correspondence, observed, "Joe is putting forward an entirely different formulation of quantum mechanics, a formulation which he is claiming is equivalent to the others and more useful in solving evolution equations, how the system evolves from time to time. Dirac didn't understand it; he didn't think it possible and he contradicts himself. But this is precisely why his [Moyal's] work and his statistical method is being used so widely today" (McIntosh, 2003).

Similarly, Dr John Corbett, emeritus professor of quantum physics at Macquarie, noted that the correspondence revealed "not only how new ideas and approaches are only accepted reluctantly, and how even very good scientists can read their own problems into another's work," but that Dirac was overly concerned with the quantization problem. "While his own method did not give a one-to-one correspondence between a classical quantity and a quantum counterpart," he concluded, "Dirac failed to yield answers

[2] No copy of the original paper survives.

and played his cards close to his chest" (Corbett, 2005).

Delivering the correspondence by email, I sought the further evaluation of several key international scientists working in the field of Moyal phase space whom I traced through the Web of Science. Their response was generous and enlightening. At the Centre of Theoretical Physics in Marseilles, Dr Bruno Iochum, fascinated to find that Moyal of the 'Moyal planes' was "a person," put me in touch with his colleagues, Drs. Joseph Várilly and José Gracia-Bondia, at the University of Costa Rica, San Jose, and the University of Complutense in Madrid respectively, who had conducted their joint research deriving from the Moyal paper. Together they contributed valuable information, checks and insights to my research. "Without dispute," wrote Várilly, "'Quantum mechanics as a statistical theory' is one of the great physical papers of the 20th century" (Várilly, 2003).

In addition, a collegiate duo of American physicists reputed in the field gladly accepted copies of the Dirac-Moyal correspondence and offered their informed judgments. Professor Thomas Curtright of the Department of Physics at the University of Miami summed up: "Seven of the letters are pure <u>gems</u>. They definitely show Dirac to be wrong about some really basic points in quantum mechanics. That by itself is most remarkable. But then they also show that Dirac is basically unfair and incredibly stubborn." "Indeed," he added, "it is stunning to a reader well-versed in quantum mechanics that Dirac — the master formalist — makes such silly mistakes and commits them in writing for posterity" (Curtright, 2003).

Concomitantly, Dr Comas Zachos of the Division of High Energy Physics at Moyal's former stamping ground, Argonne National Laboratory, had published papers featuring Moyal's work and had already recorded that his pioneering paper of 1949 had offered "a grand synthesis of the scattered mathematical machinery into a confident interpretation of quantum mechanics as a statistical theory." Writing to me with substance over several years, he set down: "Moyal's most celebrated pioneering 1949 paper in which he established an independent formulation of quantum mechanics in phase space is well validated by posterity." "This formulation of quantum mechanics serves as describing quantum transport processes in phase space. Such processes are of importance in quantum optics, nuclear and particle physics, condensed matter, the study of semi-classical limits of mesoscopic system and phase transition of classical statistical mechanics. It is the natural language to the study of quantum chaos and decoherence (of utility, e.g., in quantum computing) and provides crucial intuition in quantum mechanical interference problems, probability flows as negative probability backflows and measurements of atomic systems. The mathematical structure of the formulation is of relevance to Lie Algebras, martingales in turbulence, and string theory…In addition, it is significant outside physics, as for example in fundamental work on wavelet methods in signal processing."

I had much to learn. Surprised that Dirac "did not jump at the opportunity to embrace the innovations [that] are now seen to be compatible with this methodology," Zachos also noted that, rather, the great physicist "had declined to give ground even in the final edition of his *Principles of Quantum Mechanics* in 1958" (Zachos, 2005).

This rich polyvocal input into my research — conducted by email with quickening pace — convinced me of the importance of this case study, a study with so high a figure that captured an experience known throughout the world of science. As a historian totally untutored in physics, quantum mechanics and statistics, I profited deeply from the interest of these distinguished scientists and their friendship and zeal. The American scholars subsequently republished 'Quantum Mechanics as a Statistical Theory' in their *Quantum Mechanics in Phase Space. An Overview with Selected Papers* (Zachos, Curtright, Fairlie, 2005).

The manuscript collection, 'P. A. M. Dirac–J. E. Moyal Correspondence 1944–1946', was held by the Basser Library, Australian Academy of Science from 1962 to 2017, when, with the closure of the Library, it was transferred to the Papers of Ann Moyal, National Library of Australia, Canberra. Professor Dirac's letters are written by hand. The J. E. Moyal Medal and Lecture, established in 2000, is awarded annually at Macquarie University in mathematics, statistics and physics.

References

Bartlett, M. and Moyal, J. E., (1949), 'The exact transition probability of quantum mechanical oscillators calculated by the phase-space method', *Proceedings Cambridge Philosophical Society*, 45: 545–553.

Corbett, J., letter to Ann Moyal, 5 May 2003.

Curtright, T. L., email communication, May 2003.

Curtright, T. L., Fairlie, D. B., and Zachos, C. K., (2014), *A Concise Treatise on Quantum Mechanics in Phase Space*, Singapore, World Scientific Publishing, http://www.hep.anl.gov/czachos/a.pdf

Dirac, P. A. M., (1930–1958), *Principles of Quantum Mechanics,* Oxford University Press.

Farmelo, G., (2009), *The Strangest Man. The Hidden Life of Paul Dirac, Mystic of the Atom*. Faber and Faber, London.

Kragh, H. S., (1999), *Dirac. A Scientific Biography,* Cambridge University Press, p. 42 and *Proc. Roy. Soc. Lond***.** A113, p. 641 and p. 21

McIntosh, A., oral interview with Ann Moyal, 9 May 2003, Canberra.

Medawar, P. B., (1969), 'Two conceptions of science', *The Art of the Soluble: Creativity and Originality in Science,* Penguin, Harmondsworth, pp. 127–146.

Moyal, Ann, (2006), *Maverick Mathematician. The Life and Science of J. E. Moyal*, ANU E Press, Canberra. This contains the full published correspondence of P. A. M. Dirac and J. E. Moyal, Appendix 2. http://press.anu.edu.au/node/301/download

Moyal, Ann, oral interview with J. E. Moyal, 1979, Canberra.

Moyal, J. E., (1949a), 'Quantum mechanics as a statistical theory,' *Proceedings Cambridge Philosophical Society*, 45: 99–124, reproduced in Moyal, Ann (2006), Appendix 3.

Moyal, J. E., (1949b), 'Stochastic processes and statistical physics,' *Journal Royal Statistical Society* B, 11: 150–210.

Várilly, J., email communication to A. Moyal, April 2003.

Zachos, C. K., email communication to A. Moyal, May–June 2003 and October 2005.

Zachos, C. K., Fairlie, D. B., and Curtright, T. L. (eds.), (2005), *Quantum Mechanics in Phase Space: An Overview with Selected Papers*, World Scientific Series in 20th Century Physics, Vol. 34, World Scientific Publishing, Singapore.

Dr Ann Moyal AM was awarded the Royal Society of New South Wales Inaugural Medal in the History and Philosophy of Science in 2015.